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therefore, express the successive densities of the perihelia between sphere and sphere.

	Numbers of Perihelia.	Cubical Space.	Density of Perihelia.
0 — 20	8·65	1	8·65
20 40	11·70	7	1·67
40 60	20·30	19	1·06
60 80	17·20	37	0·47
80 100	20·80	61	0·34
100 120	8·65	91	0·095

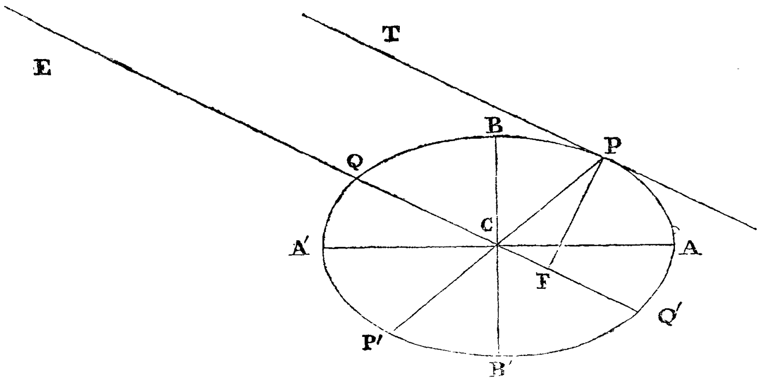
It is evident, then, that the density of the perihelia increases rapidly in approaching the sun. If the numbers in the last column of the table be compared with the inverse powers of the distance, it will be found that this increase of density is more rapid than the inverse distance, but less so than the inverse distance squared.

Two planes being supposed to be drawn, each at an angle of 45° with the plane of the ecliptic, and so that the intersection of one of them with the plane of the ecliptic is directed to the 75° of longitude, and that of the other to the 200° of longitude, there is a tendency of the cometary orbits to crowd towards the directions of these planes, not certainly so decided as to establish a law, but too pronounced to be casual.

Note on the Correction to be applied to the Apparent Ellipticity of a Planet, in consequence of the Elevation of the Earth above the Plane of the Planet's Equator. By Mr. Grant.

As the subject of the ellipticity of the planets, more especially *Saturn*, appears to excite considerable interest at present among astronomers, it may be desirable to exhibit a convenient mode of computing the effect due to the elevation of the earth above the plane of the planet's equator.

Let A B A′ B′ represent an elliptical section of the planet formed by a plane passing through the polar axis B B′ and the eye of an



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observer at the surface of the earth. Let A A′ be the equatoreal diameter situate in this plane, and let C Q E represent the direction of the observer, with respect to C, the centre of the planet. It is manifest that the angle A′ C E will then represent the elevation of the earth above the plane of the planet's equator. Let P P′ represent the extremities of the apparent polar axis of the planet, and let the direction of the visual ray at P be indicated by P T, which will evidently be a tangent at that point to the elliptical section of the planet. Moreover, since the radius C P is small compared with the distance of the earth from the planet, P T may be supposed without sensible error to be parallel to E C, and, consequently, Q Q′ will be a conjugate diameter to P P′.

Draw P F perpendicular to Q Q′. The apparent semi-polar axis will obviously be measured by the angle which this line subtends at the place of the observer on the surface of the earth.

Let $\angle$ A′ C Q $= \varphi$, P F $= p$.

By the well-known property of the ellipse, we have

$$\mathrm{PF} = \frac{\mathrm{AC} \times \mathrm{BC}}{\mathrm{CQ}}$$

$$\text{or} \qquad p = ab \,.\, \frac{\sqrt{1 - e^2 \cos^2 \varphi}}{a\sqrt{1 - e^2}}$$

$$\text{Hence} \qquad b = p \,.\, \frac{\sqrt{1 - e^2}}{\sqrt{1 - e^2 \cos \varphi}}$$

e being the *eccentricity* of the spheroid.

This expression agrees with Bessel's remark (*Ast. Nach.* vol. xii. p. 169), wherein he states that the reduction of the projection of the spheroid upon its lesser axis may be effected by multiplying the apparent polar diameter by $\dfrac{\sqrt{1 - e^2}}{\sqrt{1 - e^2 \cos^2 \varphi}}$.

It may, however, perhaps, be found convenient to present this result in the form of a *correction* to the apparent ellipticity of the planet, so as to deduce the true ellipticity from the apparent; and *vice versâ*, independently of a knowledge of the polar diameter.

$$\text{Now} \qquad p = \frac{b\sqrt{1 - e^2 \cos^2 \varphi}}{\sqrt{1 - e^2}} = a\sqrt{1 - e^2 \cos^2 \varphi}$$

$$\therefore \qquad \frac{p}{a} = \sqrt{1 - e^2}\sqrt{1 + \frac{e^2}{1 - e^2} \,.\, \sin^2 \varphi}$$

$$= \sqrt{1 - e^2} + \frac{1}{2} \,.\, \frac{e^2}{\sqrt{1 - e^2}} \,.\, \sin^2 \varphi - \frac{1}{8} \,.\, \frac{e^4}{(1 - e^2)^{\frac{3}{2}}} \,.\, \sin^4 \varphi + \text{\&c.}$$

$$\text{or} \qquad \frac{p}{a} = \frac{b}{a} + \delta,$$

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putting $\delta = \frac{1}{2} \cdot \frac{e^2}{\sqrt{1-e^2}} \cdot \sin^2\phi - \frac{1}{8} \cdot \frac{e^4}{(1-e^2)^{\frac{3}{2}}} \sin^4\phi + \&c.$

Hence $$1 - \frac{b}{a} = 1 - \frac{p}{a} + \delta$$

or $$\varepsilon = \varepsilon' + \delta,$$

where ε denotes the *true* ellipticity of the planet, and ε' the *apparent* ellipticity.

It appears, then, that δ represents the correction to be applied to the apparent ellipticity of the planet, in order to determine the true ellipticity,

Since $$\frac{p}{a} = \frac{b}{a} + \delta$$

$$\therefore \quad b = p - a\,\delta$$

In this case then $a\,\delta$ denotes the correction of the polar semidiameter.

When the question refers to the actual ellipticity of any planet, the first two terms of δ will be found quite sufficient for all practical purposes. We may, therefore, thus recapitulate the results,—

$$\varepsilon' = 1 - \frac{p}{a} \qquad (1)$$

$$\delta = \tfrac{1}{2} \cdot \frac{e^2}{\sqrt{1-e^2}} \cdot \sin^2\phi - \tfrac{1}{8} \cdot \frac{e^4}{(1-e^2)^{\frac{3}{2}}} \cdot \sin^4\phi \qquad (2)$$

$$\varepsilon = \varepsilon' + \delta \qquad (3)$$

$$b = p - a\,\delta \qquad (4)$$

The last equation will be applicable in the case wherein it is required to deduce from a series of measures of the *apparent* polar diameter made in different parts of the planet's orbit, the corresponding values of the *true* polar diameter with the view of subjecting the results to direct comparison, after they have been reduced to the mean distance from the earth.

The numerical values of the coefficients of $\sin^2\phi$, $\sin^4\phi$, in the expression for δ, may be readily determined in any particular case from any assigned value of the ellipticity. Thus, with respect to *Saturn*, if we assume with Bessel,

$$\varepsilon = 0{\cdot}098{,}048$$

we have $e = \sqrt{2\,\varepsilon - \varepsilon^2} = 0{\cdot}431845$

therefore $\delta = (9{\cdot}014{,}4440)\sin^2\phi - (7{\cdot}772{,}6766)\sin^4\phi$

and, consequently,

$$\varepsilon = \varepsilon' + (9{\cdot}014{,}4440)\sin^2\phi - (7{\cdot}772{,}6766)\sin^4\phi$$

where the numbers within the parentheses represent the logarithms of the coefficients.

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The correction will manifestly attain its maximum value when the earth has ascended to its greatest elevation above the equator of the planet. According to Bessel, the inclination of the plane of the ring (and consequently that of the equator of the planet) to the ecliptic, amounts to 28° 10′ 44″·7 — 0″·350 (t — 1800). In consequence of the latitude of the planet, when it is at a distance of 90° from the node of the ring, the maximum elevation of the earth above the equator of the planet will generally be somewhat less than this quantity. Let us suppose the maximum elevation of the earth to be 27°, we have, then,

$$\delta = 0{\cdot}021{,}055$$

$$\varepsilon' = \varepsilon - \delta = 0{\cdot}098{,}048 - 0{\cdot}021{,}055$$

$$= 0{\cdot}076{,}993 = \frac{1}{12{\cdot}9882}$$

This is, therefore, the minimum apparent ellipticity which the planet can exhibit, supposing the true ellipticity to be 0·098048.

Next to determine the ellipticity indicated by Mr. De La Rue's measures. It appears from his note that they were made on November 19, 1852. Now, if we calculate the elevation of the earth above the equator of the planet for the same day, by means of Bessel's elements of the ring (*Ast. Nach.* vol. xii., p. 170), we find it to be 20° 32′·1. Hence, using the value of e deducible from Bessel's measures, we obtain $\delta = 0{\cdot}012{,}631$, and repeating the process with the new value of e deducible from this result, we obtain for the more accurate value of the correction,

$$\delta = 0{\cdot}012{,}177.$$

Again, the mean value of his measures of the apparent polar diameter was found by him to be 18″·68, and the mean value of the equatoreal diameter to be 20″·35.

Whence $$\varepsilon' = 0{\cdot}082{,}064 = \frac{1}{12{\cdot}1856}$$

and $$\varepsilon = \varepsilon' + \delta = 0{\cdot}082{,}064 + 0{\cdot}012{,}177$$

$$= 0{\cdot}094{,}241 = \frac{1}{10{\cdot}611}$$

The corresponding value of the *apparent* ellipticity deducible from Bessel's measures, is,

$$\varepsilon' = \varepsilon - \delta = 0{\cdot}098{,}048 - 0{\cdot}012{,}631 = 0{\cdot}085{,}417.$$

The measures from which Mr. Lassell deduced the apparent ellipticity mentioned by him in his letter to Mr. De La Rue, inserted in the *Monthly Notices* for January, viz. $\frac{1}{12{\cdot}697} = 0{\cdot}078{,}764$,

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appear to have been made about the same time.* Hence, assuming the same value of ϕ, we obtain $\delta = 0{\cdot}011{,}734$, and therefore,

$$\iota = \iota' + \delta = 0{\cdot}078{,}764 + 0{\cdot}011{,}734$$
$$= 0{\cdot}090{,}498 = \frac{1}{11{\cdot}050}$$

The corresponding value of ι', deducible from Mr. Main's ellipticity, viz. $\frac{1}{9{\cdot}227}$, will be $0{\cdot}09434 = \frac{1}{10{\cdot}6}$; and the value for the same date, indicated by the ellipticity which Mr. Main has derived from the measures made by Bessel during the almost total disappearance of the ring, viz. $\frac{1}{9{\cdot}06}$, will be $0{\cdot}09608 = \frac{1}{10{\cdot}408}$.

As a wish has been expressed to have some standard of comparison in observations of this kind, the following table has been calculated for the latter part of the present year, from the values of the equatoreal and polar diameters of the planet corresponding to the mean distance from the sun (log = 0·979,6480), as deduced by Bessel from the totality of his measures, viz. 17″·053 and 15″·381 respectively. The second column in the table exhibits the elevation of the earth above the equator of the planet, which is, in the present instance, towards the *south* pole of the planet. The third and fourth columns contain the corresponding *apparent* equatoreal and polar diameters of the planet. The numbers in the fifth column give the correction to be applied to the true ellipticity in order to find the apparent ellipticity, the values of which are exhibited in the sixth and last column. The table commences with the 1st of September, when the planet will have become sufficiently favourable for observation.

Date.	ϕ	$2a$	$2b$	δ	ι'
1853.	° ′	″	″		
Sept. 1	24 53·8	18·253	16·795	0·01814	0·07991
11	24 53·1	18·590	17·104	0·01812	0·07993
21	24 51·0	18·922	17·409	0·01807	0·07998
Oct. 1	24 47·4	19·237	17·697	0·01799	0·08005
11	24 42·6	19·521	17·956	0·01788	0·08016
21	24 36·7	19·761	18·174	0·01775	0·08030
31	24 29·8	19·944	18·339	0·01760	0·08045
Nov. 10	24 22·3	20·059	18·442	0·01743	0·08062
20	24 14·5	20·098	18·475	0·01726	0·08079
30	24 6·9	20·060	18·436	0·01710	0·08096
Dec. 10	24 0·0	19·946	18·328	0·01694	0·08111
20	23 54·3	19·762	18·157	0·01682	0·08123
30	23 50·2	19·520	17·933	0·01673	0·08132

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* Mr. Lassell's measures have been communicated to the Society since the above was written, and are inserted at page 183 of the present number of the *Monthly Notices.*